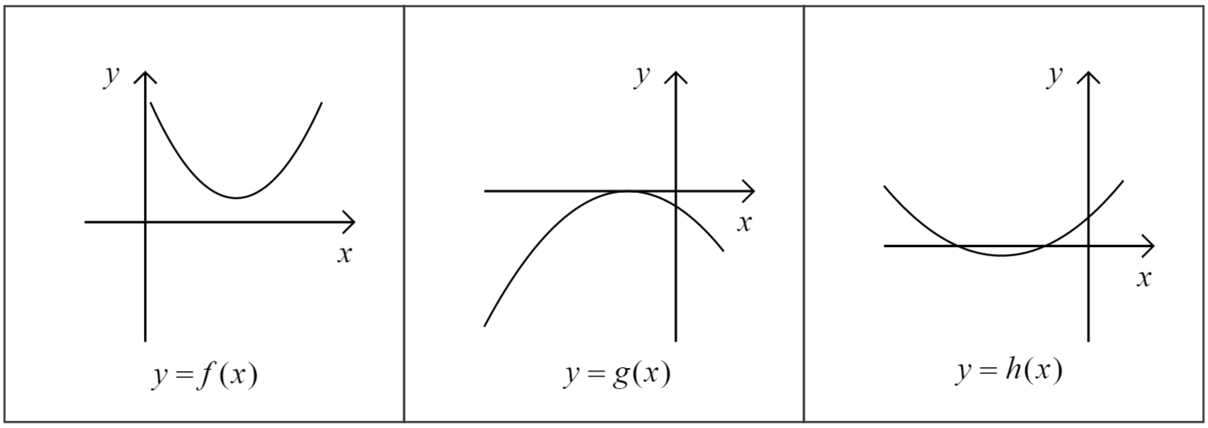
**1.** [Maximum points: 6]

The graphs below show the parabolas $y=f(x)$, $y=g(x)$ and $y=h(x)$.




Determine whether the discriminant of each of the following functions is positive, negative or zero. Give a reason for each answer.

(a) $f(x)$ [2]

(b) $g(x)$ [2]

(c) $h(x)$ [2]

**2.** [Maximum points: 6]

Consider the set of integers $\{1,1,5,49\}$.

(a) Find the value of [3]

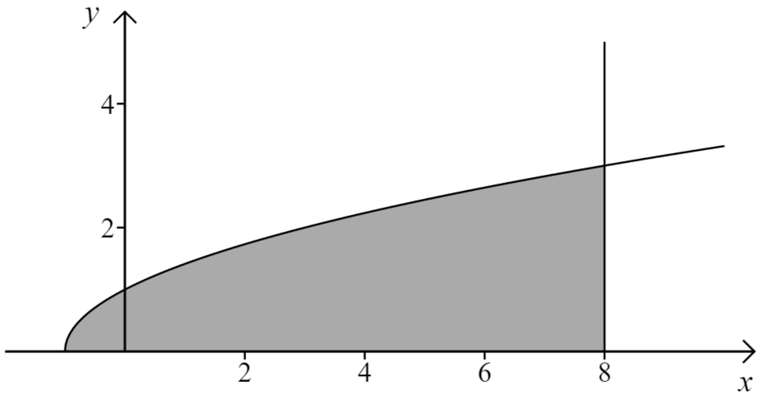
(i) the mode

(ii) the mean

(b) Find the total number of sets of four positive integers with the same mean and (single) mode as in part (a). [3]

**3.** [Maximum points: 6]

The graph below shows the area completely bound by the curve $y = \sqrt{x+1}$, the $x$-axis and the line $x = 8$.

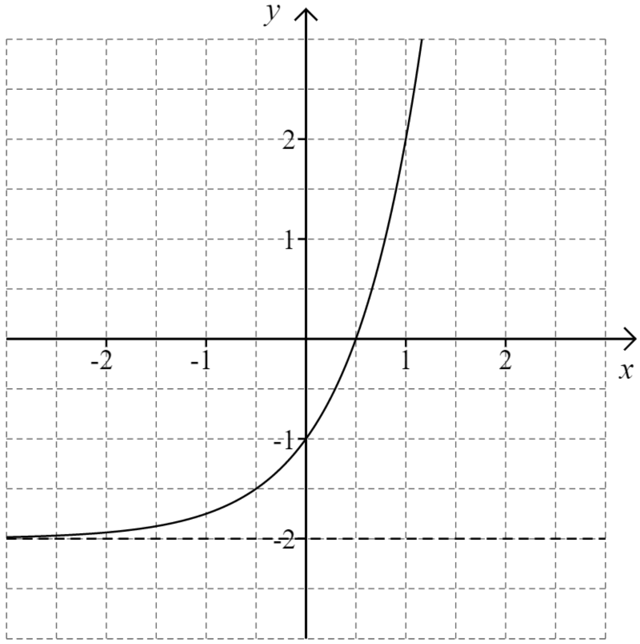



(a) Find the $x$-intercept of the curve $y = \sqrt{x+1}$. [2]

(b) Find the area of the shaded region. [4]

**4.** [Maximum points: 6]

Let $f(x) = 4^x - 2$. The diagram below shows the graph of $y = f(x)$.




(a) Sketch the graph of $y = f^{-1}(x)$ on the axes above. [3]

(b) Find the function $f^{-1}(x)$. [3]

**5.** [Maximum points: 6]

The following table shows values of the functions $f$ and $g$ for various values of $x$.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $f(x)$ | 4 | 2 | 1 | 0 | 3 |
| $g(x)$ | 1 | 2 | 3 | 4 | 0 |

Solve the following equations

(a) $(f \circ g)(x) = 4$ [3]

(b) $(g \circ g)(x) = 3$ [3]

**6.** [Maximum points: 8]

Let $f(x) = \frac{\sin x}{1 + \sin x}$.

(a) Find $f'(x)$. [3]

(b) Determine whether $f(x)$ is increasing or decreasing when $\pi/2 < x < 3\pi/2$. [2]

(c) Find the equation of the tangent line to the graph of $y = f(x)$ when $x = \pi$. [3]

**7.** [Maximum points: 14]

A regular six-sided die is repeatedly rolled until a 5 appears. Let $X$ represent the total number of rolls of the die.

(a) Describe the sample space of $X$. [1]

(b) **Copy and complete** the following table showing the first four values of $X$ and their probabilities. [4]

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $P(X=x)$ | | | | |

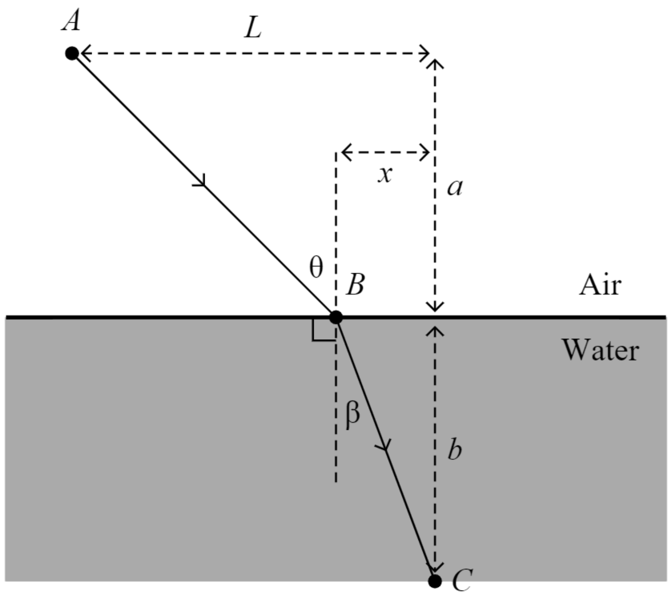
(c) Find an expression for $E(X)$ using sigma notation. [2]

(d) Show that $E(X) - \frac{5}{6} \cdot E(X) = \frac{1}{6}\left(1 + \sum_{k=1}^{\infty}\left(\frac{5}{6}\right)^{k}\right)$. [3]

(e) Hence find the value of $E(X)$. [4]

**8.** [Maximum points: 15]

A beam of light travels through the air in a straight line at $c$ m/s, where $c$ is a constant, from point $A$ to point $B$ where it meets a body of water. It enters the water and travels in a straight line at $0.75c$ m/s to point $C$. This is shown in the diagram below.




(a) In terms of $L$, $x$ and $a$ write down an expression for [3]

(i) length $AB$

(ii) $\sin\theta$

(b) In terms of $x$ and $b$ write down an expression for [2]

(i) length $BC$

(ii) $\sin\beta$

(c) Show that the time $t$ for the light beam to travel from point $A$ to point $C$ via point $B$ is given by [3]

$$t = \frac{\sqrt{(L-x)^2 + a^2}}{c} + \frac{4\sqrt{x^2 + b^2}}{3c}$$

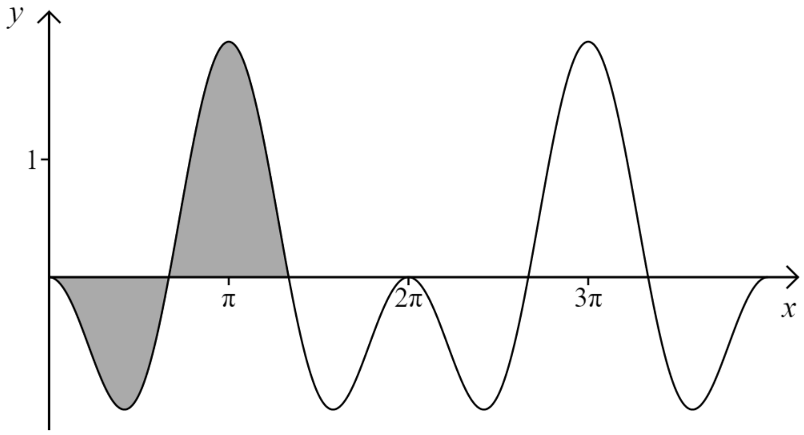
(d) Find $\frac{dt}{dx}$. [4]

The beam of light travels from point $A$ to point $C$ via point $B$ using the path which takes the shortest amount of time.

(e) Show that this path satisfies the equation $\frac{\sin\theta}{\sin\beta} = \frac{4}{3}$. [3]

**9.** [Maximum points: 13]

Let $f(x) = 2\cos^2 x - \cos x - 1$. The diagram below shows the graph of $y = f(x)$. Part of the region between the graph and the $x$-axis has been shaded.




(a) By using the substitution $u = \cos x$ or otherwise solve the equation $f(x) = 0$. [5]

(b) Use a suitable trigonometric identity to show that [3]

$$\int f(x)\,dx = \frac{\sin 2x}{2} - \sin x + C$$

where $C \in \mathbb{R}$.

(c) Hence find the area of the shaded region. [5]

**1.** (a) The graph doesn't cross the $x$-axis so $f(x) = 0$ has no real solutions. R1

The discriminant is therefore negative. A1

(b) The graph touches the $x$-axis once so $g(x) = 0$ has a repeated root. R1

The discriminant is therefore zero. A1

(c) The graph crosses the $x$-axis twice so $h(x) = 0$ has two solutions. R1

The discriminant is therefore positive. A1

**2.** (a)

(i) 1 A1

(ii) $\frac{56}{4} = 14$ M1A1

(b) The first two values must be 1. A1

The sum of the third and fourth value must be 54. A1

The total number of ways this can happen (while having a single mode) is 26. A1

**3.** (a) We have

$$0 = \sqrt{x+1} \qquad \text{M1}$$

So $x = -1$. A1

(b) The area is given by

$$\int_{-1}^{8} \sqrt{x+1}\, dx = \frac{2}{3}\left[(x+1)^{3/2}\right]_{-1}^{8} \qquad \text{M1A1}$$

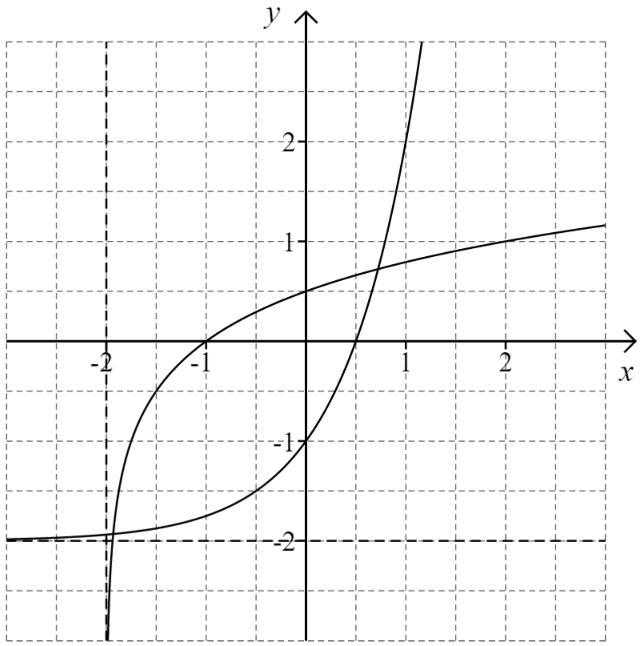
This is equal to

$$\frac{2}{3} \cdot \left(\sqrt{9}\right)^3 = \frac{54}{3} \qquad \text{M1A1}$$

**4.** (a) The axes intercepts are $(0, 1/2)$ and $(-1, 0)$. A1

There is a vertical asymptote at $x = -2$. A1

The shape is approximately correctly. A1




(b) We have

$$x = 4^y - 2 \quad \text{M1}$$

So

$$y = \log_4(x+2) \quad \text{A1}$$

Therefore

$$f^{-1}(x) = \log_4(x+2) \quad \text{A1}$$

**5.** (a) We have

$$f(0)=4 \quad \text{M1}$$

So

$$g(x)=0 \quad \text{A1}$$

Giving $x=4$. A1

(b) We have

$$g(2)=3 \quad \text{M1}$$

So

$$g(x)=2 \quad \text{A1}$$

Giving $x=1$. A1

**6.** (a) Use the quotient rule M1

$$f'(x) = \frac{\cos x(1+\sin x) - \cos x \sin x}{(1+\sin x)^2} = \frac{\cos x}{(1+\sin x)^2}$$ A1A1

(b) Since $f'(x) < 0$ the function is decreasing. M1A1

(c) The gradient is $f'(\pi) = -1$. A1

So we have

$$y - 0 = -(x - \pi)$$ M1

This gives

$$y = -x + \pi$$ A1

**7.** (a) Positive integers A1

(b) We have

$$P(X=1)=\frac{1}{6}$$ A1

$$P(X=2)=\frac{5}{6}\times\frac{1}{6}=\frac{5}{36}$$ A1

$$P(X=3)=\left(\frac{5}{6}\right)^2\times\frac{1}{6}=\frac{25}{216}$$ A1

$$P(X=4)=\left(\frac{5}{6}\right)^3\times\frac{1}{6}=\frac{125}{1296}$$ A1

(c) $E(X)=\sum_{k=1}^{\infty}\frac{k}{6}\cdot\left(\frac{5}{6}\right)^{k-1}$ A1A1

(d) We have

$$\frac{5}{6}\cdot E(X)=1\cdot\frac{5}{6}\cdot\frac{1}{6}+2\cdot\left(\frac{5}{6}\right)^2\cdot\frac{1}{6}+3\cdot\left(\frac{5}{6}\right)^3\cdot\frac{1}{6}+\cdots$$ A1

So

$$E(X)-\frac{5}{6}\cdot E(X)=1\cdot\frac{1}{6}+(2-1)\cdot\frac{5}{6}\cdot\frac{1}{6}+(3-2)\cdot\left(\frac{5}{6}\right)^2\cdot\frac{1}{6}+\cdots$$ M1

This gives

$$E(X)=\frac{1}{6}+\frac{1}{6}\sum_{k=1}^{\infty}\left(\frac{5}{6}\right)^k=\frac{1}{6}\left(1+\sum_{k=1}^{\infty}\left(\frac{5}{6}\right)^k\right)$$ A1

(e) Simplify the left side, and rewrite the right side using the formula for an infinite geometric series M1

$$\frac{E(X)}{6}=\frac{1}{6}\left(1+\frac{5/6}{1-5/6}\right)=\frac{1}{6}(1+5)=1$$ A1A1

So

$$E(X)=6$$ A1

**8.** (a)

(i) $|AB| = \sqrt{(L-x)^2 + a^2}$ A1A1

(ii) $\sin\theta = \dfrac{L-x}{\sqrt{(L-x)^2 + a^2}}$ A1

(b)

(i) $|BC| = \sqrt{x^2 + b^2}$ A1

(ii) $\sin\beta = \dfrac{x}{\sqrt{x^2 + b^2}}$ A1

(c) Use time = distance $\div$ speed. We have M1

$$t = \frac{\sqrt{(L-x)^2 + a^2}}{c} + \frac{\sqrt{x^2 + b^2}}{0.75c} = \frac{\sqrt{(L-x)^2 + a^2}}{c} + \frac{4\sqrt{x^2 + b^2}}{3c}$$ A1A1

(d) Use the chain rule M1

$$\frac{dt}{dx} = \frac{2x - 2L}{2c\sqrt{(L-x)^2 + a^2}} + \frac{8x}{6c\sqrt{x^2 + b^2}} = \frac{x - L}{c\sqrt{(L-x)^2 + a^2}} + \frac{4x}{3c\sqrt{x^2 + b^2}}$$ A1A1A1

(e) We have

$$0 = \frac{x - L}{c\sqrt{(L-x)^2 + a^2}} + \frac{4x}{3c\sqrt{x^2 + b^2}}$$ A1

Therefore

$$0 = -\frac{\sin\theta}{c} + \frac{4\sin\beta}{3c}$$ M1

Therefore

$$\frac{\sin\theta}{\sin\beta} = \frac{4}{3}$$ A1

**9.** (a) Factorise

$$(2u+1)(u-1)=0 \qquad \text{M1}$$

So

$$u=-1/2$$

or

$$u=1 \qquad \text{A1}$$

Giving

$$x=\frac{2\pi}{3}+2n\pi \qquad \text{A1}$$

or

$$x=\frac{4\pi}{3}+2n\pi \qquad \text{A1}$$

or

$$x=2n\pi \qquad \text{A1}$$

(b) Use a double angle identity M1

$$\int f(x)\,dx=\int \cos 2x-\cos x\,dx \qquad \text{A1}$$

This is equal to

$$\frac{\sin 2x}{2}-\sin x+C \qquad \text{A1}$$

(c) The area of the first region is

$$\left|\frac{\sin(4\pi/3)}{2}-\sin(2\pi/3)-\frac{\sin 0}{2}+\sin 0\right|=\frac{3\sqrt{3}}{4} \qquad \text{M1A1}$$

The area of the second region is

$$\frac{\sin(8\pi/3)}{2}-\sin(4\pi/3)-\frac{\sin(4\pi/3)}{2}+\sin(2\pi/3)=\frac{3\sqrt{3}}{2} \qquad \text{M1A1}$$

The total area is therefore $\frac{9\sqrt{3}}{4}$. A1