1. *Level 1 − 2* *[Length: 3 minutes]*

Solve the following equations:

(a) $(x-3)(x+4)=0$ [1]

(b) $x(x-7)=0$ [1]

(c) $-3(1-x)(x+4)=0$ [1]

2. *Level 1 − 2* *[Length: 6 minutes]*

For each of the following parabolas (i) write the equation in the form $y = (x - p)^2 + q$ where $p, q \in \mathbb{Z}$ and (ii) write down the coordinates of the vertex.

(a) $y = x^2 + 4x + 5$ [3]

(b) $y = x^2 - 6x - 1$ [3]

3. *Level 1 − 2 [Length: 4 minutes]*

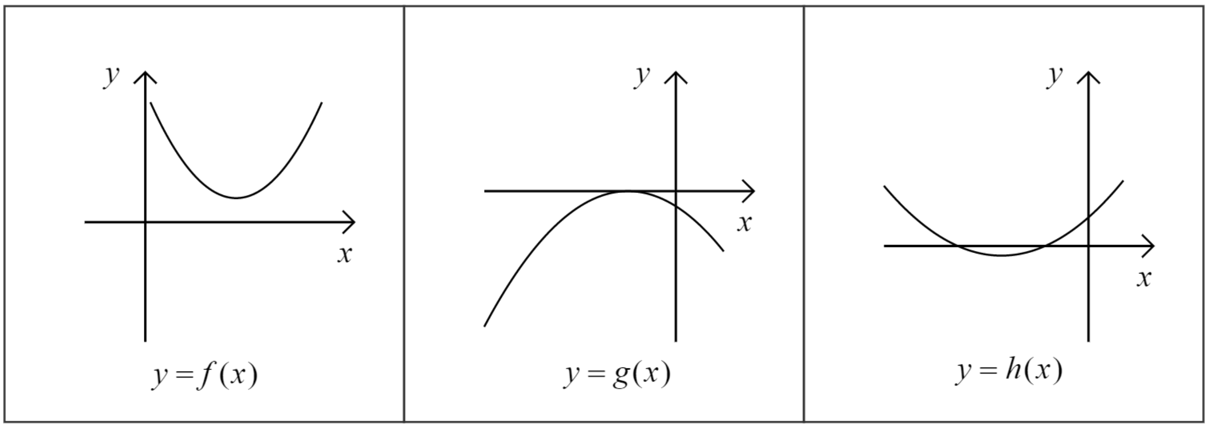
Solve the following equations:

(a) $x^2 - 2x - 8 = 0$ [2]

(b) $x^2 - 16x = 0$ [2]

4. *Level 3 − 4 [Length: 6 minutes]*

The graphs below show the parabolas $y = f(x)$, $y = g(x)$ and $y = h(x)$.



Determine whether the discriminant of each of the following functions is positive, negative or zero. Give a reason for each answer.

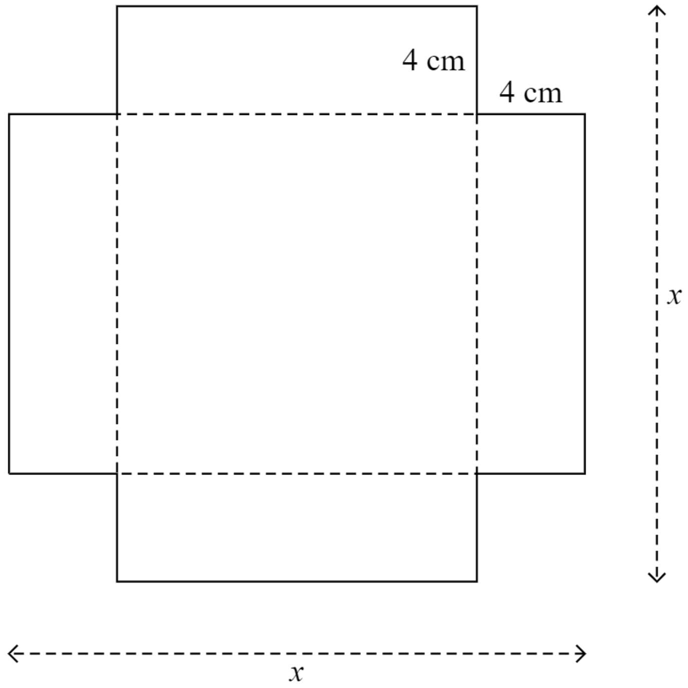
(a) $f(x)$ [2]

(b) $g(x)$ [2]

(c) $h(x)$ [2]

5. *Level 3 − 4* *[Length: 6 minutes]*

A square sheet of card has sides of length x cm. Squares with sides of length 4 cm are removed from each corner of the card. This is shown in the diagram below.



The four edges of the card are folded up by 90° to form a box with a square base.

(a) Write down the height of the box. [1]

(b) Find the length and width of the base of the box in terms of x. [1]

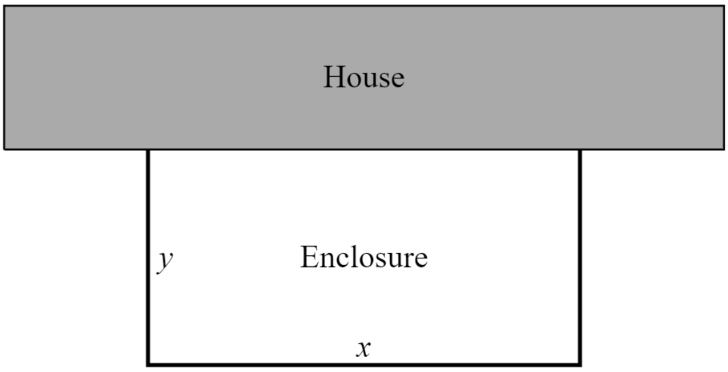
The volume of the box is 196 cm^2.

(c) Use this information to write down an equation involving x. [1]

(d) Solve the equation to determine the possible value(s) of x. [3]

6. *Level 5 − 6 [Length: 7 minutes]*

A farmer wishes to build a rectangular enclosure next to his house. He has 40 m of fencing with which he can do this. He only needs to build three sides of the enclosure as shown in the diagram below.



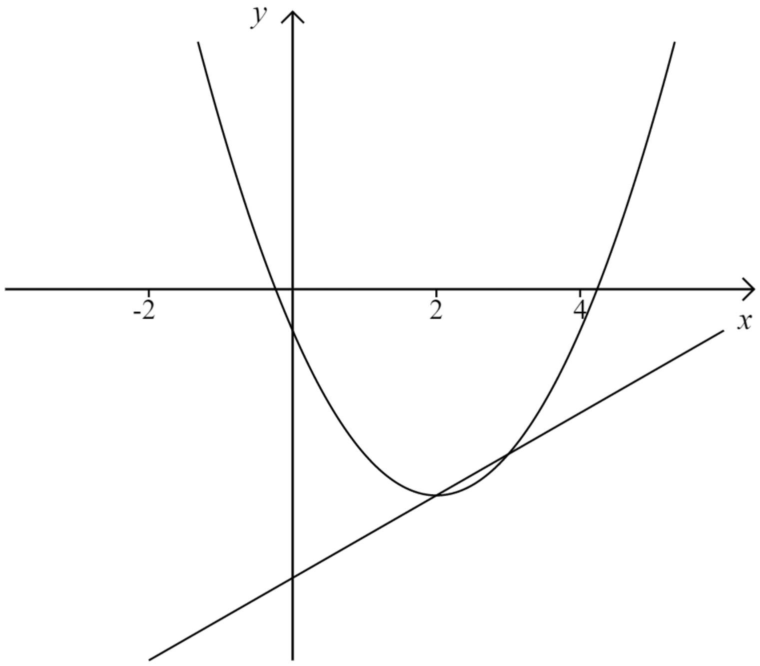
Let the width of the enclosure be x and the height be y.

(a) Show that the area A of the enclosure is equal to $-\frac{x^2}{2}+20x$. [4]

(b) Determine the values of x and y of the enclosure with the largest area. [3]

7. *Level 5 − 6 [Length: 6 minutes]*

The diagram below shows the parabola $y = x^2 - 4x - m$ and the line $y = mx + c$ where $m, c \in \mathbb{R}$.



The x-coordinates of the points of intersection of the parabola and the line are $x = 2$ and $x = 3$. Find the values of m and c.

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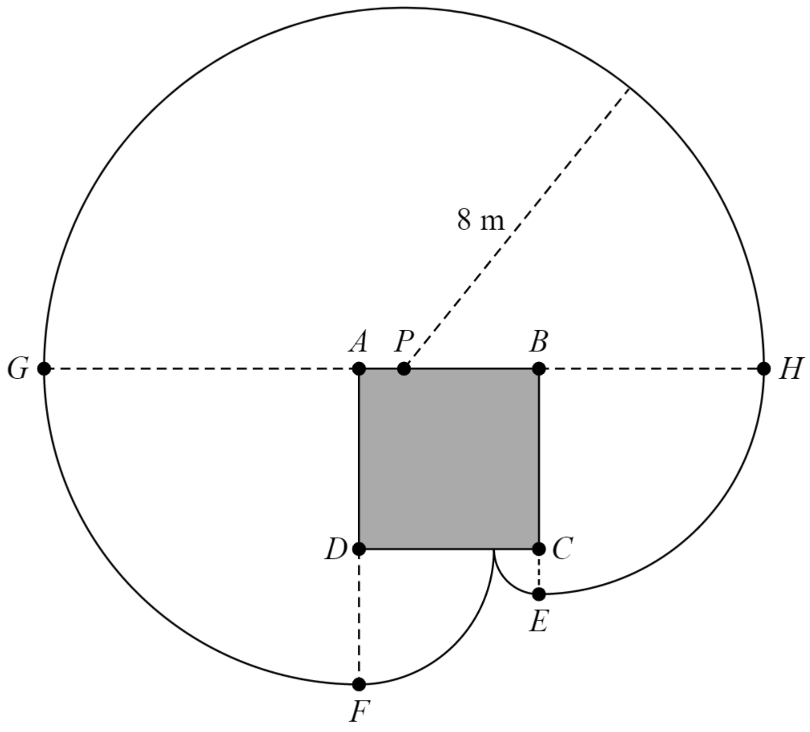
8. *Level 7 − 8 [Length: 13 minutes]*

A square shed with sides of length 4 m sits in the middle of a large garden. A dog is attached to point P with a lead of length 8 m. This is shown in the diagram below where the solid curve represents the boundary of the area where the dog is free to roam.



Let $AP = x$ where $0 \le x \le 4$.

(a) Find the following lengths in terms of x [3]

(i) AG

(ii) BH

(b) Show that $DF = 4 - x$ and $CE = x$. [2]

(c) Find the total area A where the dog is free to roam in terms of x. Expand your answer. [3]

(d) Sketch the graph of the function from part (c). [3]

(e) Find the value(s) of x that maximise the area where the dog is free to roam. [2]

1. (a) 3 or −4

(b) 0 or 7

(c) −4 or 1

2. (a)

(i) $y = (x + 2)^2 + 1$

(ii) $(-2, 1)$

(b)

(i) $y = (x - 3)^2 - 10$

(ii) $(3, -10)$

3. (a) Factorise

$$(x-4)(x+2)=0$$

So

$$x = -2 \text{ or } 4$$

(b) Factorise

$$x(x-16)=0$$

So

$$x=0 \text{ or } 16$$

4. (a) The graph doesn't cross the x-axis so $f(x) = 0$ has no real solutions.

The discriminant is therefore negative.

(b) The graph touches the x-axis once so $g(x) = 0$ has a repeated root.

The discriminant is therefore zero.

(c) The graph crosses the x-axis twice so $h(x) = 0$ has two solutions.

The discriminant is therefore positive.

5. (a) 4 cm

(b) $x - 2 \times 4 = x - 8$

(c) $4(x - 8)^2 = 196$

(d) We have

$$(x - 8)^2 = 49$$

So

$$x - 8 = \pm 7$$

Giving

$$x = 1 \text{ or } 15$$

However, we can reject $x = 1$ since x must be greater than 8. So $x = 15$ cm.

6. (a) There is 40 m of fence so $x + 2y = 40$.

The area A of the enclosure is equal to xy.

Rearrange $x + 2y = 40$ and substitute into $A = xy$.

$$A = x \times \frac{40 - x}{2}$$

Which can be rewritten as $A = -\frac{x^2}{2} + 20x$.

(b)

We have

$$A = -\frac{1}{2}(x - 20)^2 + 200$$

So $x = 20$.

This gives $y = 10$.

(This question can also be solved by differentiating)

7. We have

$$2^2 - 4(2) - m = 2m + c$$

And

$$3^2 - 4(3) - m = 3m + c$$

Simplifying gives

$$-4 - 3m = c$$

And

$$-3 - 4m = c$$

Solve simultaneously e.g.

$$-4 - 3m = -3 - 4m$$

So

$$m = 1$$

And

$$c = -7$$

8. (a)

(i) $8-x$

(ii) $8-(4-x)=4+x$

(b) We have

$$DF=8-x-4=4-x$$

And

$$CE=4+x-4=x$$

(c) Use the area of a circle formula

$$A=\frac{\pi\times 8^2}{2}+\frac{\pi(8-x)^2}{4}+\frac{\pi(4+x)^2}{4}+\frac{\pi(4-x)^2}{4}+\frac{\pi x^2}{4}$$

This simplifies to

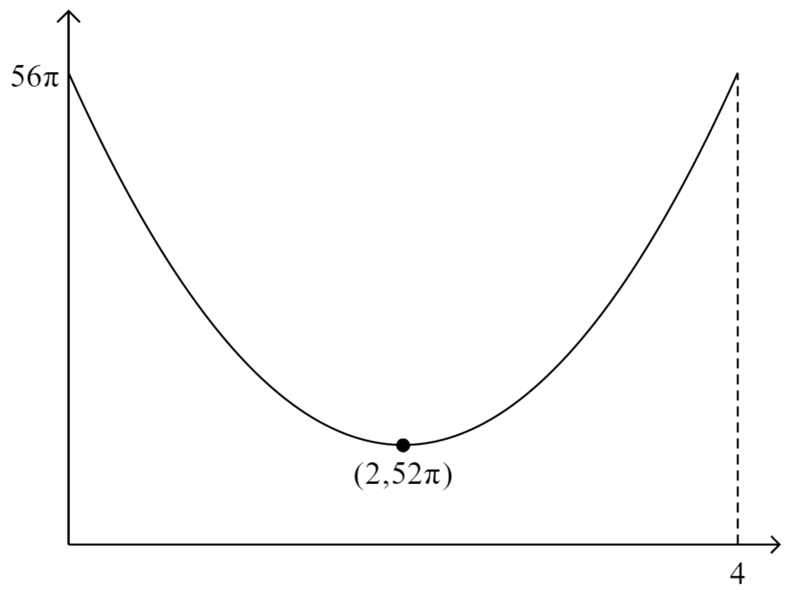
$$A=\pi x^2-4\pi x+56\pi$$

(d) Rewrite in vertex form

$$A=\pi(x-2)^2+52\pi$$

The domain is $x\in[0,4]$.

The y-intercept and vertex are clearly labelled.



(e) The dog's area is maximised when $x=0$ or 4.

He is a good boy.