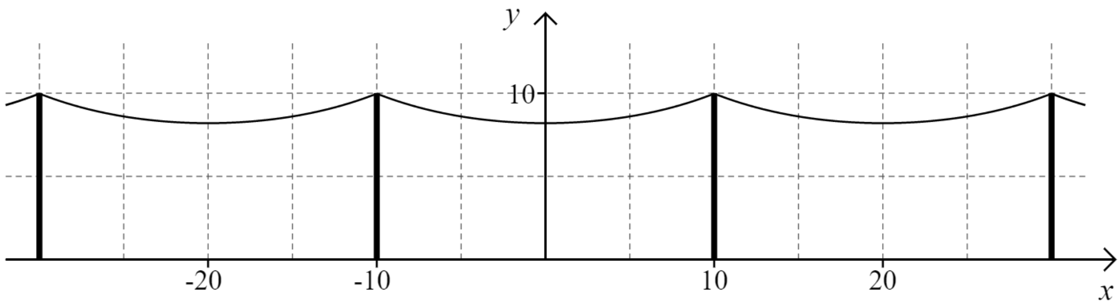
1. [Maximum points: 12]

Telegraph poles 10 m in height each seperated by 20 m hold up a length of cable. The length of cable between each pole is equal to 20.5 m. This is shown in the diagram below.

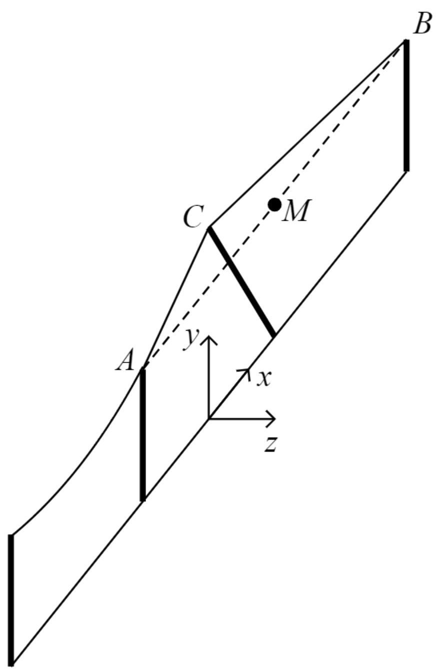


The cable between the middle two poles can be modelled by $y = e^{x/8} + e^{-x/8} + a$ where $a \in \mathbb{R}$.

(a) Find the value of a. [2]

(b) Find the height of the lowest point of the cable above the ground. [2]

A truck crashes into the pole at $x = 10$ resulting in the pole leaning at an angle. The cable between this pole and the two adjacent poles becomes taut, resulting in the cable forming two straight lines. This is shown in the diagram below where point M is the midpoint of AB.



(c) Write down the length of AB. [1]

(d) Calculate length CM. [2]

(e) Find the angle between the damaged pole and the ground. [5]

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2. [Maximum points: 18]

A soft drink manufacturer makes bottles of a certain soft drink which is labelled as containing 500 ml of drink. The actual amount in each bottle is normally distributed with a mean of 507 ml and a standard deviation of 2 ml.

(a) In a random sample of 10,000 bottles calculate the expected number of bottles that fail to contain 500 ml of drink. [3]

Let the random variable X represent the amount of drink in a randomly selected bottle.

(b) If $P(507 - x < X < 507 + x) = 0.99$ calculate the value of x. [3]

(c) Hence **copy and complete** the following sentence: [1]

The amount of drink in 99% of bottles is within .... of the mean.

The manufacturer notices that one of its bottling machines has been operating inconsistently. It takes a random sample of bottles created by the machine and measures the amount of drink in each. The results are shown in the table below.

Amount of drink (a)	Frequency
$a \leq 503$	53
$503 < a \leq 505$	128
$505 < a \leq 507$	302
$507 < a \leq 509$	290
$509 < a \leq 511$	98
$a > 511$	23

A χ^2 goodness of fit test is performed at the 10% significance level to determine if the distribution has changed.

(d) Write down [2]

(i) the null hypothesis

(ii) the alternate hypothesis

(e) State the degrees of freedom for the test. [1]

(f) Calculate the expected frequencies. [4]

(g) Find the p-value for the test. [2]

(h) State the conclusion of the test. Give a reason for your answer. [2]

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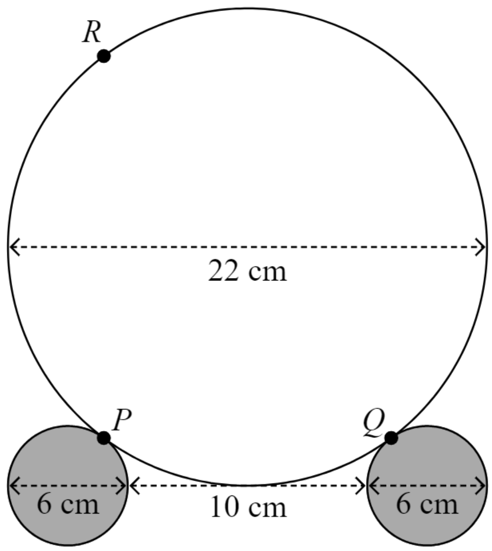
3. [Maximum points: 11]

In a bowling alley balls of diameter 22 cm are returned to the player by rolling along a track consisting of two metal cyclindrical tubes of diameter 6 cm. The tubes are separated by a gap of 10 cm.

A cross section of the tubes and the ball is shown below. The tubes are in contact with the ball at points P and Q. Point R is directly above point P giving $\angle QPR = 90°$.



(a) Find the distance from the centre of a tube to the centre of the ball. [2]

(b) Show that $PQ = \frac{88}{7}$ cm. [3]

(c) Find the value of length PR to four decimal places. [3]

As the ball travels along the tubes it makes 5 rotations per second.

(d) Find the speed of the ball in m/s. [3]

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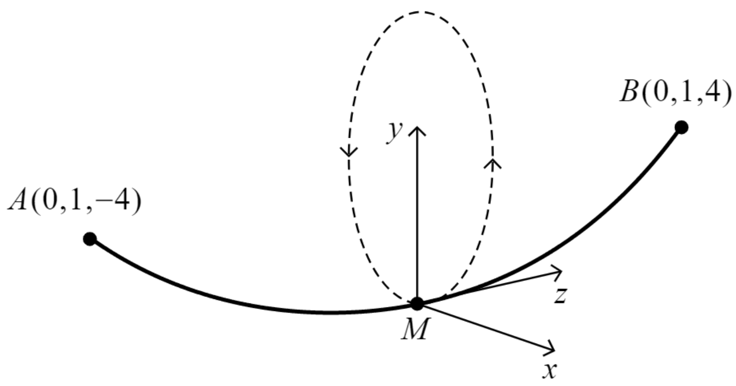
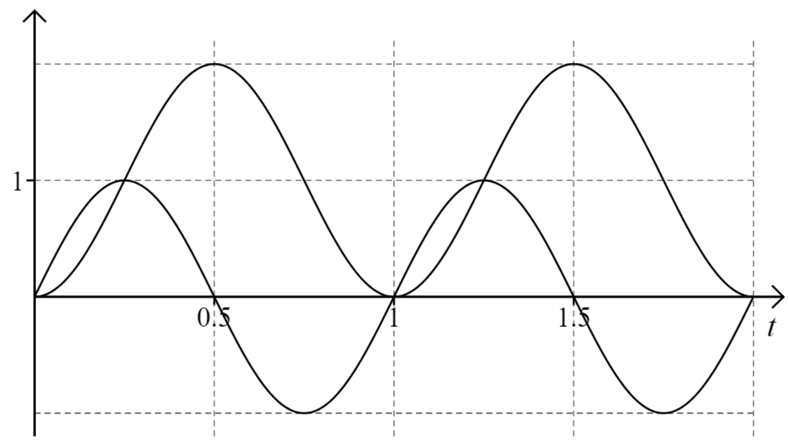
4. [Maximum points: 18]

Two children hold a rope at points $A(0,1,-4)$ and $B(0,1,4)$ where units of coordinates are metres. They swing the rope so that its midpoint M moves in a complete circle every second. Initially point M is at the origin $(0,0,0)$. This is shown in the diagram below.



(a) Write down the distance between the endpoints of the rope. [1]

The x-coordinate of point M after t seconds is given by the function $x(t) = a\sin(bt°)$ and the y-coordinate is given by $y(t) = c\cos(bt°) + d$ where $a, b, c, d \in \mathbb{R}$. The diagram below shows the graphs of these functions.



(b) Write down the value of [3]

(i) a

(ii) c

(iii) d

(c) Find the value of b. [2]

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Another child stands with their feet at the origin and jumps vertically over the rope each time point M approaches the bottom of the circle. In order to jump over the rope without hitting it, the child jumps before point M is closer than 80 cm to the origin.

(d) In the interval $[0.5, 1.5]$ find the range of values of t for which point M is closer than 80 cm to the origin. Write your answer to four significant figures. [4]

The height $h(t)$ above ground of the child's feet t seconds after jumping is given by the equation $h(t) = ut - 5t^2$ where $u \in \mathbb{R}$.

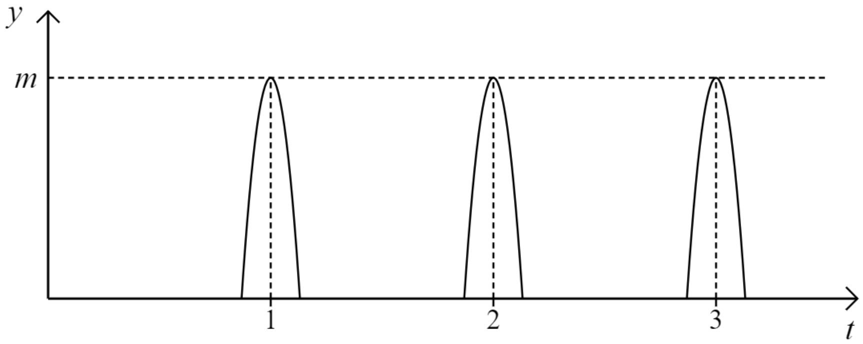
(e) Using the rounded values of t from part (d) show that the smallest possible value of u so that the child is able to jump over the rope without hitting it is 1.31. [3]

For the remainder of the question you may assume $u = 1.31$.

Let the maximum height of the child's feet above the ground during a jump be equal to m.

(f) Find the value of m. [2]

The graph below shows the y-coordinate of the child's feet during the first 3.5 seconds.



(g) Write down the equations of the three parabolas. [3]

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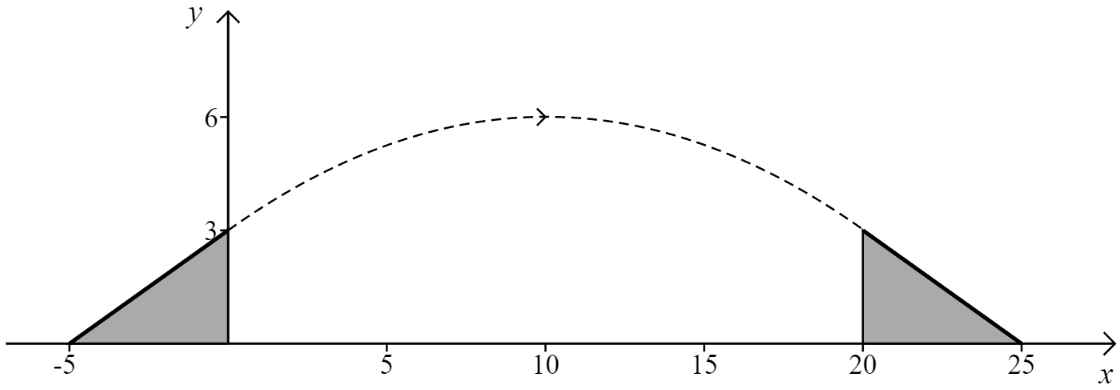
5. [Maximum points: 19]

A stunt motorcyclist jumps off of a ramp and lands on an identical ramp situated 20 m away. The path of the motorcylist through the air is a parabola. Both ramps are tangential to the parabola. This is shown in the diagram below.



(a) Calculate the gradient of the ramp off of which the motorcyclist jumps. [1]

The equation of the parabola is of the form $y = ax^2 + bx + c$ where $a,b,c \in \mathbb{R}$.

(b) Write down the value of c. [1]

(c) Use the coordinates of the landing point to show that $20a + b = 0$. [2]

(d) Find $\frac{dy}{dx}$. [1]

(e) Hence show that $a = -0.03$ and $b = 0.6$. [2]

The parabola can also be written in the form

$$y = \frac{x}{\tan\theta} - \frac{5x^2}{(v\cos\theta)^2} + c$$

where c takes the value from part (b), v is the speed of the motorcyclist in m/s at the beginning of the jump, and θ is the angle of the ramp to the horizontal.

(f) Find the values of v and θ. [3]

The problem continues on the next page

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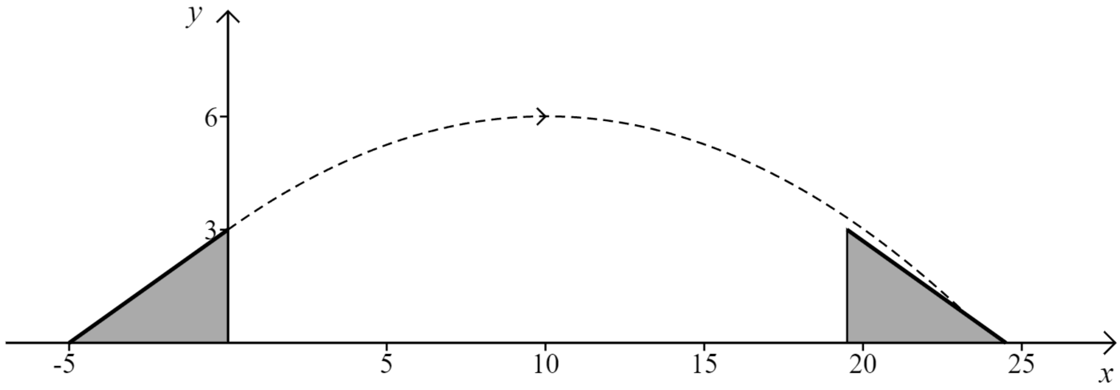
For safety reasons the landing ramp is moved 0.5 m forward. This is shown in the diagram below.



(g) Determine the equation of the line formed by the sloped part of the landing ramp in the form $y = mx + c$ where $m, c \in \mathbb{R}$. [3]

(h) If the motorcylist follows the path of the original parabola find [6]

(i) the coordinates of the landing point

(ii) the angle between the path of the motorcyclist and the landing ramp at this point

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1. (a) We have

$$e^{10/8} + e^{-10/8} + a = 10 \qquad \text{M1}$$

So

$$a = 6.22 \qquad \text{A1}$$

(b) $1 + 1 + 6.22 = 8.22\,\text{m}$ M1A1

(c) 40 m A1

(d) We have

$$CM^2 + 20^2 = 20.5^2 \qquad \text{M1}$$

So

$$CM = 4.5\,\text{m} \qquad \text{A1}$$

(e) Use the cosine rule to determine the angle to the vertical. M1

$$4.5^2 = 10^2 + 10^2 - 2 \cdot 10 \cdot 10 \cdot \cos\theta \qquad \text{A1}$$

So

$$\theta = 26.0058° \qquad \text{A1}$$

The angle to the horizontal is therefore

$$90 - 26.0058 = 64.0° \qquad \text{M1A1}$$

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2. (a) We have $P(X < 500) = 0.000232673$. M1A1

So the expected number of bottles is 2.33. A1

(b) Using invNorm with e.g. $P(X < 507 - x) = 0.005$ we have M1

$$507 - x = 501.848$$ A1

So

$$x = 5.15$$ A1

(c) The amount of drink in 99% of bottles is within 5.15 ml of the mean. A1

(d)

(i) The distribution is the same. A1

(ii) The distribution is different. A1

(e) 5 A1

(f) Use binomcdf for the given intervals and multiply by the total of the frequencies. This gives M1

20.3, 121, 305, 305, 121, 20.3 A1A1A1

(g) 2.39×10^{-11} A1A1

(h) Since $2.39 \times 10^{-11} < 0.1$ we reject the null hypothesis. So the distribution is different. R1A1

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3. (a) $\dfrac{22}{2}+\dfrac{6}{2}=14\,\text{cm}$ M1A1

(b) Use similarity M1

$$\frac{PQ}{11}=\frac{6+10}{14}$$ A1

Giving

$$PQ=\frac{11\times 16}{14}=\frac{176}{14}=\frac{88}{7}$$ A1

(c) Use the Pythagorean theorem M1

$$\left(\frac{PR}{2}\right)^2+\left(\frac{44}{7}\right)^2=11^2$$ A1

Giving

$$PR=18.0543\,\text{cm}$$ A1

(d) As the ball rotates once points P and Q trace out a path on the ball which is 18.0543π cm in length. A1

So the speed is

$$\frac{18.0543\pi\times 5}{100}=2.84\,\text{m/s}$$ M1A1

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4. (a) 8 m A1

(b)

(i) 1 A1

(ii) −1 A1

(iii) 1 A1

(c) We need

$$\frac{360}{b} = 1$$ M1

So

$$b = 360$$ A1

(d) Use the distance equation M1

$$\sqrt{(\sin(360t))^2 + (-\cos(360t) + 1)^2} < 0.8$$ A1

Solve using a GDC. This gives $t \in [0.8690, 1.131]$. M1A1

(e) The child needs to be in the air for $1.131 - 0.8690 = 0.262\,\text{sec}$. A1

So we need

$$0 = 0.262u - 5 \times 0.262^2$$ A1

Therefore $u = 1.31$. A1

(f) Determine the maximum value of $h(t) = 1.31t - 5t^2$ e.g. by using a GDC. M1

This gives 0.0858 m. A1

(g) $y = -5(t-1)^2 + 0.0858$ A1

$y = -5(t-2)^2 + 0.0858$ A1

$y = -5(t-3)^2 + 0.0858$ A1

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5. (a) $\frac{3}{5} = 0.6$ A1

(b) 3 A1

(c) We have

$$3 = 400a + 20b + 3$$ M1

So

$$400a + 20b = 0$$ A1

Giving

$$20a + b = 0$$

(d) $\frac{dy}{dx} = 2ax + b$ A1

(e) We have

$$0.6 = 2a(0) + b$$ A1

Giving

$$b = 0.6$$

So

$$20a + 0.6 = 0$$ A1

Giving

$$a = -0.03$$

(f) We have

$$\theta = \arctan 0.6 = 31.0°$$ M1

And

$$-\frac{5}{(v\cos 30.96376)^2} = -0.03$$ M1

So

$$v = 15.1 \text{ m/s}$$ A1

(g) The gradient is -0.6. A1

The equation is therefore

$$y - 0 = -0.6(x - 24.5)$$ M1

Giving

$$y = -0.6x + 14.7$$ A1

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(h)

(i) Solve $-0.6x + 14.7 = -0.03x^2 + 0.6x + 3$ e.g. by using a GDC. M1

This gives

$$x = 23.162$$ A1

So the coordinates are

$$(23.2, 0.803)$$ A1

(ii) The gradient of the tangent to the parabola is

$$2 \times (-0.03) \times 23.162 + 0.6 = -0.7897$$ A1

So the angle between the tangent and the slope is

$$|\arctan(-0.7897)| - \arctan 0.6 = 7.34°$$ M1A1

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