1. [Maximum points: 4]

A regular six-sided die is rolled 1000 times. The results are shown below.

Outcome (X)	$0 < X \leq a$	$a < X \leq b$	$b < X \leq c$	$c < X \leq 6$
Frequency	316	173	341	170

Determine the likely values of a, b and c.

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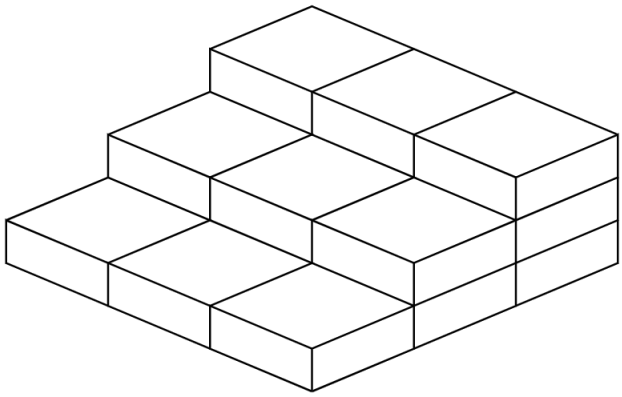
2. [Maximum points: 4]

Some steps are to be built using identical bricks each in the shape of a square prism. The following diagram shows how 18 bricks can be used to build 3 steps.



Calculate how many bricks are needed to build 20 steps.

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3. [Maximum points: 4]

In this problem write all answers to two decimal places.

The value of a motorbike depreciates at a rate of 25% per year. The price when new is \$4500.

(a) Determine the price of the motorbike after five years. [2]

The rate of inflation is 4%.

(b) Determine the real value of the motorbike after five years. [2]

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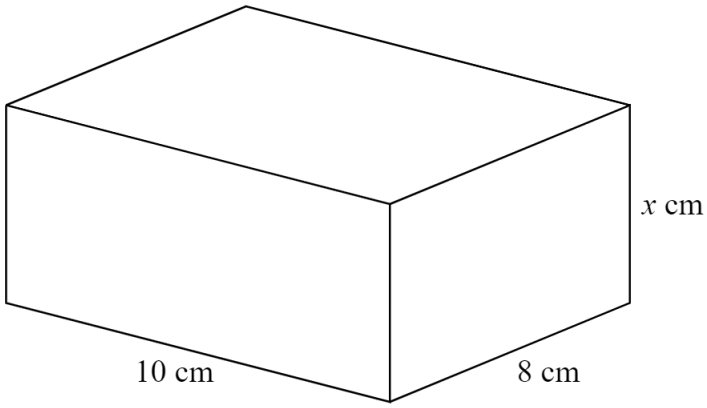
4. [Maximum points: 5]

A solid wooden rectangular prism has dimensions x cm $\times$ 10 cm $\times$ 8 cm where $x \in \mathbb{N}$. This is shown in the diagram below.



The cube is cut in half producing two identical rectangular prisms. The total surface area of these new prisms is 120 cm^2 greater than the surface area of the original prism.

(a) Add to the diagram above showing the direction of the cut. Clearly explain your reasoning for the direction. [3]

(b) Calculate the value of x. [2]

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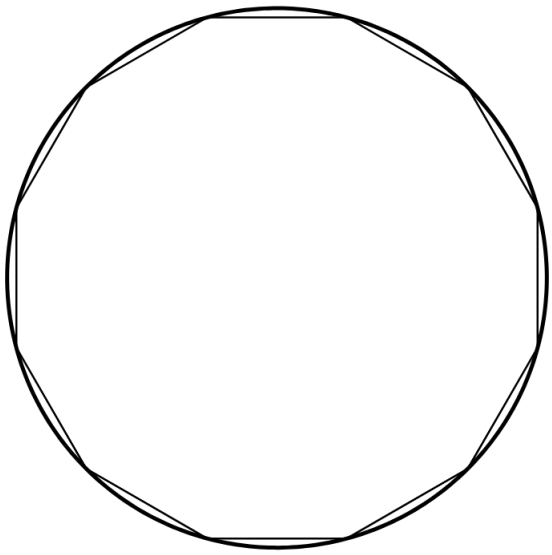
5. [Maximum points: 5]

The diagram shows the largest regular dodecagon which fits inside a circle of radius 5 cm.



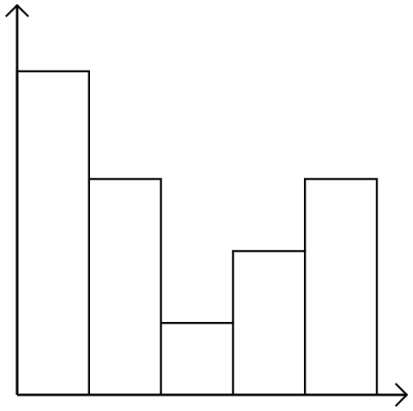
Calculate the area of the dodecagon.

6. [Maximum points: 5]

The test for a particular disease is 95% accurate. It is known that 10% of the population have the disease. A member of the population is selected at random. Determine the probability this member has the disease given that the test indicates so.

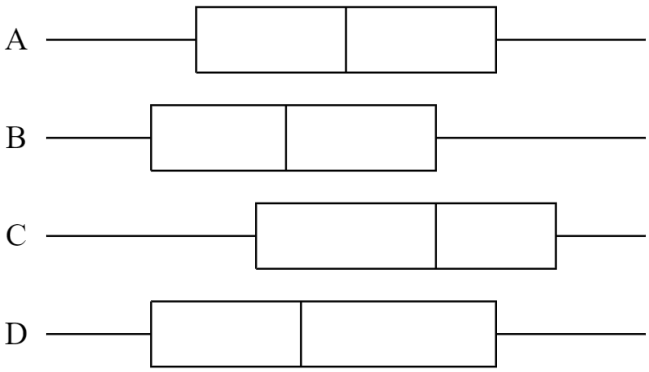
7. [Maximum points: 6]

A school wishes to determine how many hours of TV each student watches per day. They survey the first 100 people to enter the dining room at lunchtime. The results are shown in the frequency histogram below.

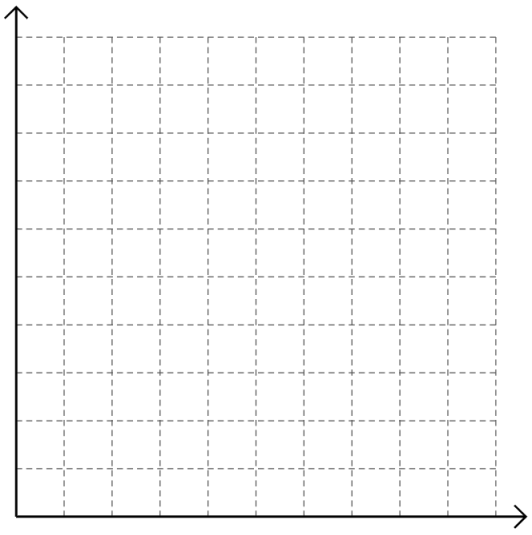


(a) Write down the sampling method used. [1]

(b) Choose the box-and-whisker plot below that best fits the data above. [2]



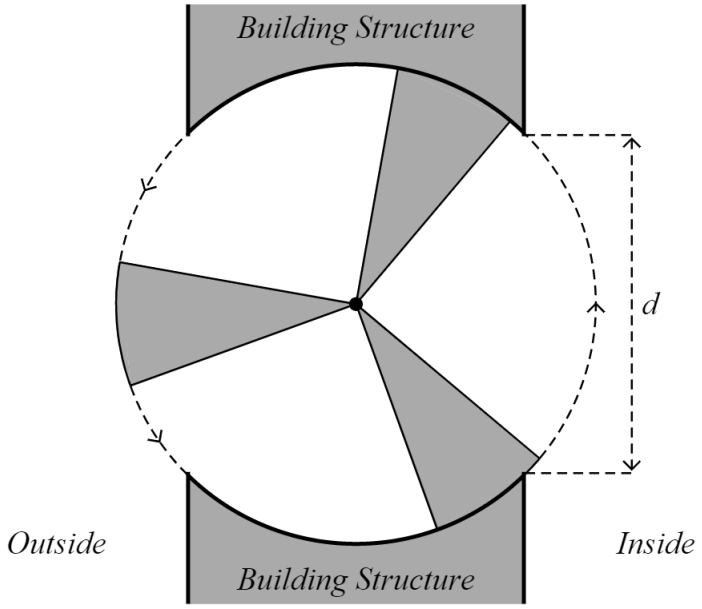
(c) On the axes below sketch a cumulative frequency curve for this data. [3]



8. [Maximum points: 6]

An office building has a revolving door of radius 2 m at the entrance. The door contains three compartments each of equal area through which people may walk as the door revolves. The compartments are seperated by three circular segments each of equal area.

The diagram below shows the view of the door from above. The door rotates in an anti-clockwise direction.



The angle subtended by each compartment is three times the angle subtended by each circular segment.

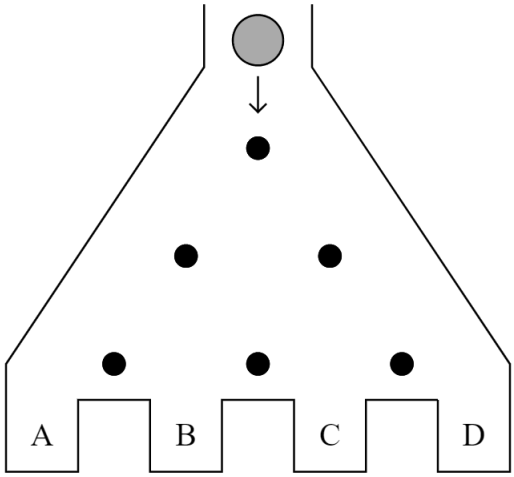
(a) Show that each compartment forms a right-angle at the centre of the door. [3]

The doors are designed so that if they stop rotating then there is no gap between the inside and the outside of the building through which wind can blow through unobstructed.

(b) Find the largest possible value of d. [3]

9. [Maximum points: 6]

A ball is dropped into the machine below which contains six pins. The ball hits a pin on each row and has an equal chance of going left or right. The ball then lands in one of four cups.



(a) Complete the following table showing the number of ways of reaching each cup. [2]

Cup	A	B	C	D
Routes	LLL		RRL RLR LRR	
Total	1		3	

(b) Complete the following table showing the probability of landing in each cup. [2]

Cup	A	B	C	D
Probability	$\frac{1}{8}$			

(c) If 1600 balls are dropped into the machine determine how many you would expect to land in cup C. [2]

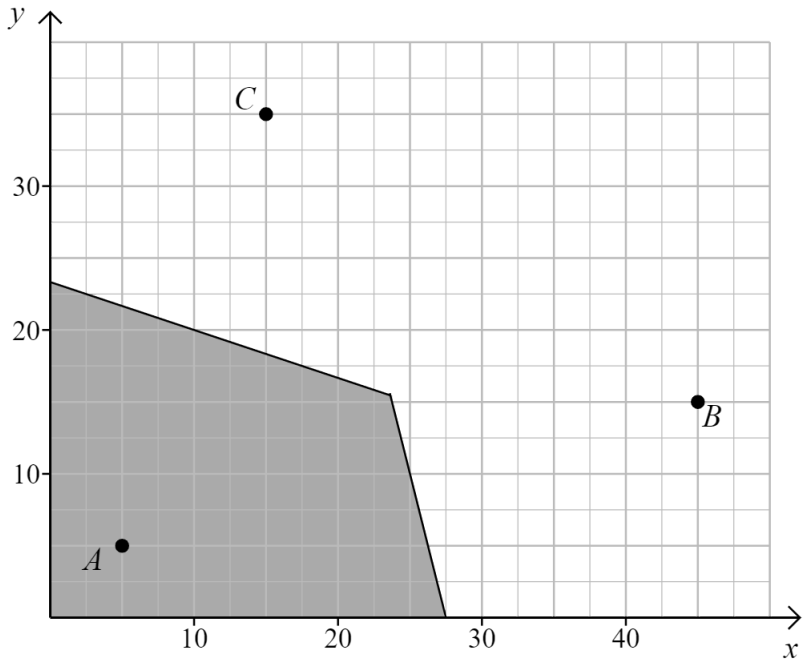
10. [Maximum points: 6]

For every hour Lee studies mathematics he increases his next mathematics test score by 9 points. Let this number of hours equal x. If Lee improves his understanding of mathematics then he also improves his understanding of science, so for every hour he studies science he increases his next science test score by $8 + x$ points.

Lee has tests in both subjects tomorrow, but he only has three hours to study for them. Determine how long he should study each subject in order to maximise the sum of his test scores.

11. [Maximum points: 6]

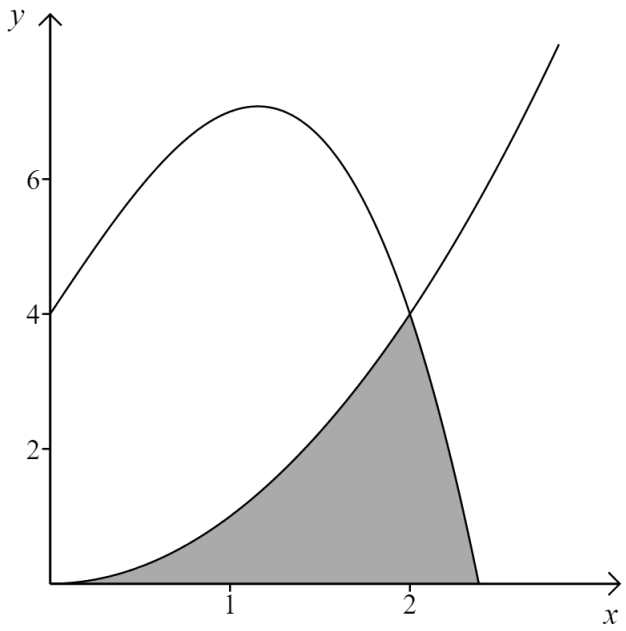
The graph below shows points $A(5,5)$, $B(45,15)$ and $C(15.35)$. The region containing all points which are the closest to point A is shaded.



On the diagram above show the region which contains all points which are the closest to point C

12. [Maximum points: 6]

Let $f(x) = -x^3 + 4x + 4$ and $g(x) = x^2$. The area of the region bound by the graphs of $y = f(x)$ and $y = g(x)$ and the x-axis in the first quadrant is shown in the diagram below.



Determine the area of the shaded region.

13. [Maximum points: 9]

The following table shows the scores of ten divers from three different judges in an international diving competition.

Diver	A	B	C	D	E	F	G	H	I	J
Judge 1	9.1	8.8	7.6	9.6	8.2	8.5	9.0	7.2	8.1	8.8
Judge 2	9.2	8.8	7.5	9.4	8.0	8.6	8.9	7.3	8.0	9.0
Judge 3	9.0	8.7	7.3	9.4	8.4	8.3	9.2	7.4	8.3	9.1

(a) Complete the following table showing the ranking of the divers from each judge. [3]

Diver	A	B	C	D	E	F	G	H	I	J
Judge 1	2									
Judge 2			9							
Judge 3						7.5				

(b) Find the Spearman's rank correlation coefficient between [4]

(i) Judge 1 and Judge 2

(ii) Judge 2 and Judge 3

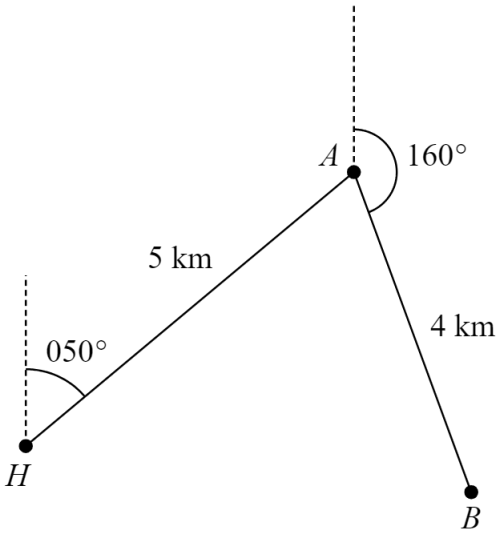
(iii) Judge 3 and Judge 1

Judges are highly trained and experienced. It is expected that each diver will receive similar scores from every judge. One of the judges is suspected of being biased when awarding points to divers from certain regions.

(c) Based on your answers to part (b) write down which judge you would expect this to be. Give a reason for your answer. [2]

14. [Maximum points: 8]

A woman leaves her home at point H and walks for 5 km on a bearing of 050° to point A. She then walks for 4 km on a bearing of 160° to point B. This is shown in the diagram below.



(a) Find the size of $\angle HAB$. [2]

(b) Calculate length HB. [2]

(c) Find the bearing at which the woman should walk from point B to point H. [4]

1. We can expect to get each value approximately $1000 \div 6 \approx 167$ times. R1

So $a = 2$, $b = 3$ and $c = 5$. A1A1A1

2. The first step has 3 bricks, the second has 6 and the third has 9. A1

This is an arithmetic sequence with first term 3 and common ratio 3. A1

The total number of bricks is therefore

$$S_{20} = \frac{20}{2}(2 \times 3 + 19 \times 3) = 630$$ M1A1

3. (a) $4500 \times 0.75^5 = \$1067.87$. M1A1

(b) We have

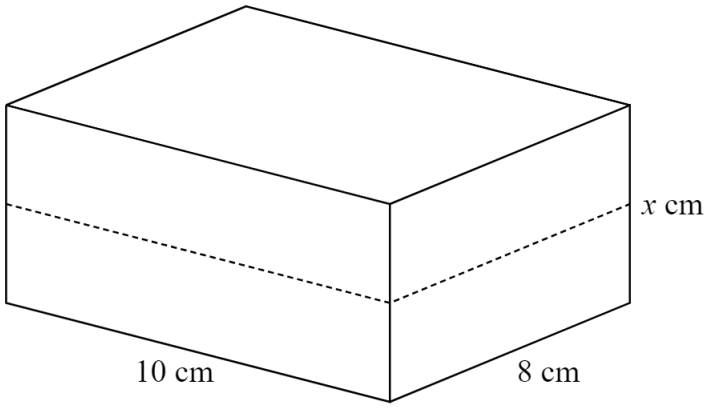
$$x \times 1.04^5 = 1067.87$$ M1

Giving

$$x = \$877.71$$ A1

4. (a) The two new surfaces will each have an area of $\frac{120}{2} = 60\,\text{cm}^2$. A1

Since only 10 is a factor of this number each new surface must have dimensions $x\,\text{cm} \times 10\,\text{cm}$. A1



A1

(b) We have

$$10x = 60 \quad \text{M1}$$

Giving

$$x = 6\,\text{cm} \quad \text{A1}$$

5. Divide the dodecagon into 12 isosceles triangles which each subtent an angle of $360/12 = 30°$ at the centre. M1

The area of one of these triangles is then

$$\frac{5 \times 5 \times \sin 30}{2} = 6.25 \quad \text{M1A1}$$

So the total area is

$$12 \times 6.25 = 75\,\text{cm}^2 \quad \text{M1A1}$$

6. Use the conditional probability formula. M1

Let D represent the event that the member has the disease.

Let d represent the event that the test indicates the member has the disease.

$$P(D \mid d) = \frac{P(D \cap d)}{P(d)} = \frac{0.1 \times 0.95}{0.1 \times 0.95 + 0.9 \times 0.05}$$ A1 A1A1

So the probability is 0.679. A1

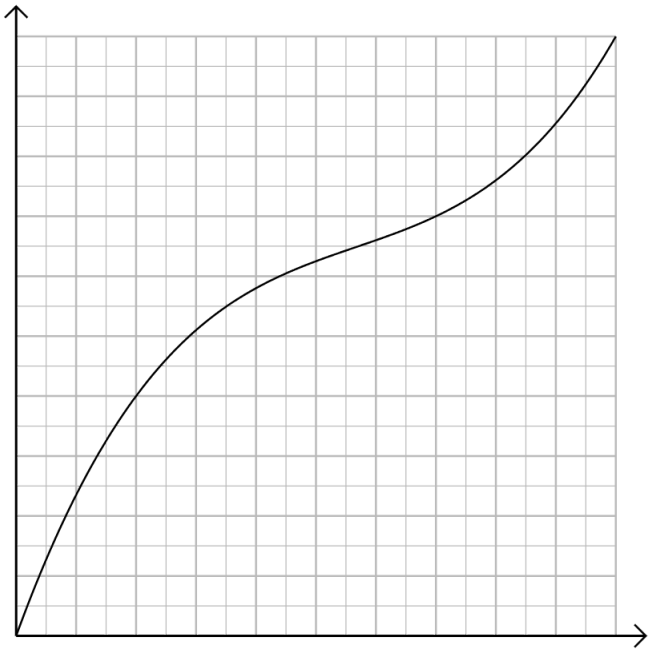
7. (a) Convenience sampling A1

(b) D A1A1

(c) The graph is concave down then concave up. A1

The gradient near the start is greater than the gradient near the end. A1

The graph is increasing. A1



8. (a) We have

$$3 \times 4\theta = 360 \qquad \text{M1}$$

Giving

$$\theta = \frac{360}{12} = 30 \qquad \text{A1}$$

So each compartment subtends an angle of $3 \times 30 = 90°$. A1

(b) Use right-angled trigonometry or the Pythagorean theorem e.g. M1

$$d = 2 \times 2\sin 45 = 2.83\,\text{m} \qquad \text{A1A1}$$

9. (a) A1A1

Cup	A	B	C	D
Routes	LLL	LLR LRL RLL	RRL RLR LRR	RRR
Total	1	3	3	1

(b)

Cup	A	B	C	D
Probability	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

A1A1

(c) $\frac{3}{8} \times 1600 = 600$ M1A1

10. If Lee studies mathematics for x hours then he can study science for $3 - x$ hours. A1

The sum of the points S he will increase his test scores by is

$$S = 9x + (8 + x)(3 - x)$$ M1

This simplifies to

$$S = -x^2 + 4x + 24$$ A1

Find the maximum point using any method e.g.

$$S = -(x - 2)^2 + 28$$ M1

So $x = 2$. A1

He should study mathematics for two hours and science for one hour. A1

11. Determine the perpendicular bisector of line BC using any method e.g.

The midpoint of line BC is $\left(\frac{15+45}{2}, \frac{35+15}{2}\right) = (30,25)$. A1

The gradient of line BC is $\frac{35-15}{15-45} = -\frac{2}{3}$. A1

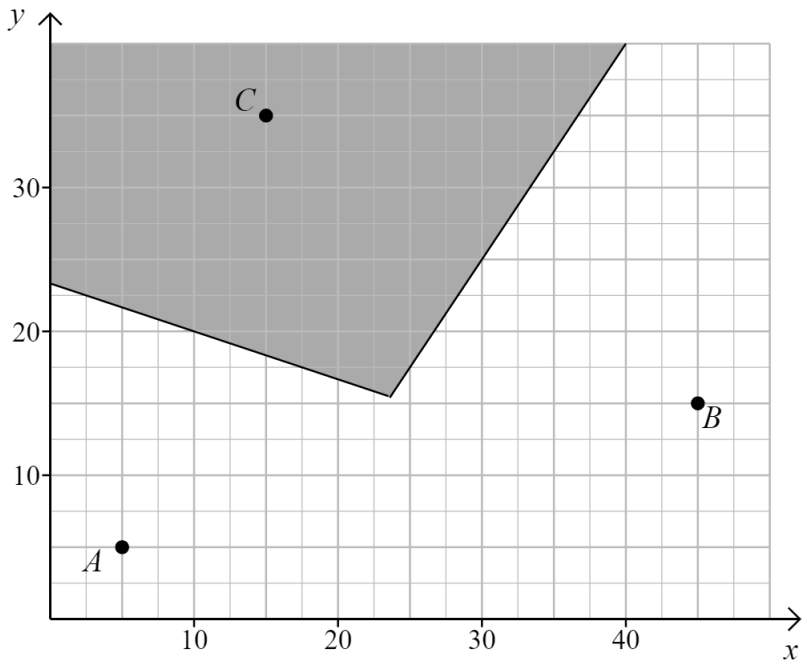
So the equation is

$$y - 25 = \frac{3}{2}(x - 30)$$ M1

Giving

$$y = \frac{3}{2}x - 20$$ A1

Add this line to the graph and shade the region shown below. A1A1



12. Use a GDC to determine the x-coordinate of the point of intersection and the x-intercepts. M1

The x-coordinate of the point of intersection is 2. A1

The x-intercepts are 0 and 2.3830. A1

Divide the area into two regions. M1

The area is then

$$\int_0^2 x^2\,dx + \int_2^{2.3830} -x^3 + 4x + 4\,dx = 3.49 \quad \text{A1A1}$$

13. (a) A1A1A1

Diver	A	B	C	D	E	F	G	H	I	J
Judge 1	2	4.5	9	1	7	6	3	10	8	4.5
Judge 2	2	5	9	1	7.5	6	4	10	7.5	3
Judge 3	4	5	10	1	6	7.5	2	9	7.5	3

(b) Use a GDC to determine each value M1

(i) 0.976 A1

(ii) 0.912 A1

(iii) 0.921 A1

(c) Judge 3 since the Spearman's rank correlation coefficient is smaller for calculations involving this judge. A1R1

14. (a) $50 + 20 = 70°$ M1A1

(b) Use the cosine rule M1

$$HB^2 = 25 + 16 - 40\cos 70$$

So

$$HB = 5.23\,\text{km}$$ A1

(c) Use the sine or cosine rule to find $\angle ABH$. M1

$$\angle ABH = \arccos\left(\frac{5^2 - 4^2 - 5.23^2}{-2 \cdot 4 \cdot 5.23}\right) = 64.0°$$ A1

So the bearing is $180 + 160 - 64 = 276°$. M1A1